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Context Effects in the Judgment of Visual Relative-Frequency: Trial-by-Trial Adaptation and Non-linear Sequential Effect

Xiangjuan Ren[1,2]*, Muzhi Wang*[3] *and Hang Zhang*[2,3,4]*

[1] *Academy for Advanced Interdisciplinary Studies, Peking University, Beijing, China,* [2] *Peking-Tsinghua Center for Life Sciences, Peking University, Beijing, China,* [3] *School of Psychological and Cognitive Sciences and Beijing Key Laboratory of Behavior and Mental Health, Peking University, Beijing, China,* [4] *PKU-IDG/McGovern Institute for Brain Research, Peking University, Beijing, China*

Humans' judgment of relative-frequency, similar to their use of probability in decision-making, is often distorted as an inverted-*S*-shape curve—small relative-frequency overestimated and large relative-frequency underestimated. Here we investigated how the judgment of relative-frequency, despite its natural reference points (0 and 1) and stereotyped distortion, may adapt to the environmental statistics. The task was to report the relative-frequency of black (or white) dots in a visual array of black and white dots. We found that participants' judgment was distorted in the typical inverted-*S*-shape, but the distortion curve was influenced by both the central tendency and spread of the distribution of objective relative-frequencies: the lower the central tendency, the higher the overall judgment (contrast effect); the higher the spread, the more curved the inverted-*S*-shape (curvature effect). These context effects are in the spirit of efficient coding but opposite to what would be predicted by Bayesian inference. We further modeled the context effects on the level of individual trials, through which we found not only a trial-by-trial adaptation, but also the non-linear sequential effects that were recently reported mainly in circularly distributed visual stimuli.

Keywords: probability distortion, subjective probability, frequency estimation, sequential effect, adaptation, Bayesian inference, efficient coding

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Correspondence:
Hang Zhang
hang.zhang@pku.edu.cn

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INTRODUCTION

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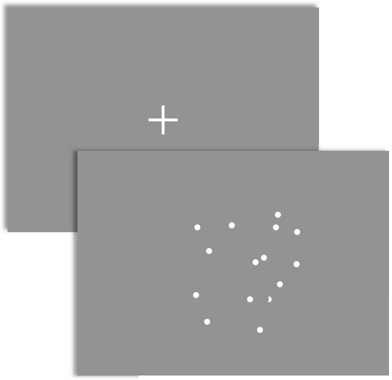

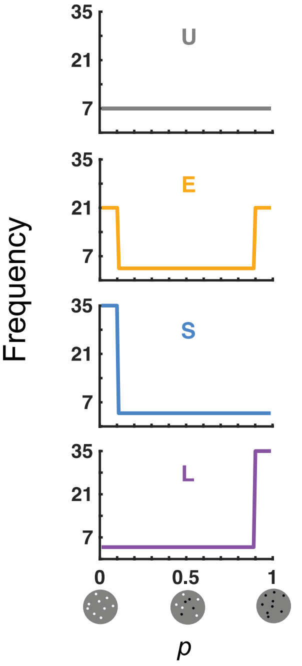
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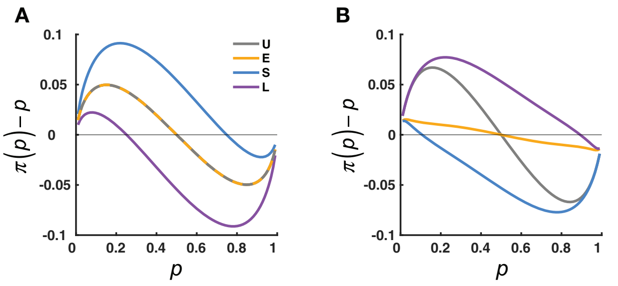


FIGURE 2 | Opposing effects predicted by two influential lines of theories**.** The predicted deviation of the subjective from objective relative-frequency, $\pi(p) - p$, is plotted as a function of the objective relative-frequency, p, and compared across the four distribution conditions (U, E, S, L, color coded). **(A)** Adaptation-level theory (Helson, 1947). The $\pi(p)$ is assumed to reflect the [illegible] p [illegible] y

$$\pi_y = \sum_q q \Pr(q \mid y)$$

[illegible] y [illegible] $\pi(p)$

$$\pi(p) = \int \pi_y \Pr(y \mid p)\, dy$$

For **Figure 2B** [illegible] $\sigma_{noise} =$ [illegible]

Adaptation-Level Theory

[illegible] S [illegible] $\pi(p)$ [illegible] L

$$\lambda[\pi(p)] = \gamma(\lambda[p] - \lambda[L]) + (1 - \gamma)\lambda[p]$$

[illegible] γ [illegible] p [illegible] λ [illegible] L [illegible] p

$$\lambda[L] = \eta \sum_p \theta(p)\lambda[p]$$

[illegible] η [illegible] $\theta(p)$ [illegible] Eq [illegible]

For **Figure 2A** [illegible] $\gamma =$ [illegible] $p =$ [illegible] $\eta =$ [illegible]

Measures of Distortion of Relative-Frequency

Slope and Crossover Point Estimated From LLO

For [illegible] Eq [illegible] $\pi(p)$ [illegible] γ [illegible] p

Smoothed Distortion Curve and Non-parametric Measures

[illegible] γ [illegible] p [illegible] $\pi(p) - p$ [illegible]

$$\hat{M}_h(x) = \frac{\sum_{i=1}^{m} K\left(\frac{x - x_i}{h}\right) y_i}{\sum_{i=1}^{m} K\left(\frac{x - x_i}{h}\right)}$$

[illegible]

r σ_k no p n o n rn n
o p_j n p_{j-1} r p no o r
r q nc o j n $j-1$ $j=$ N n

$$W = \begin{pmatrix} w(p_n\ p_{n-1}) & & \cdots \\ & w\ (p_n\ p_{n-1}) & \cdots \\ & & \\ & & \cdots\ w_N\ (p_n\ p_{n-1}) \end{pmatrix},$$

$$X = \begin{pmatrix} S & R & \\ S & R & \\ & & \\ S_N & R_{N-1} & \end{pmatrix},$$

$$Y = \begin{pmatrix} R \\ R \\ \\ R_N \end{pmatrix},$$

co n o q r r r on
$(p_n\ p_{n-1})$ co

$$\begin{pmatrix} \hat{\beta}^{WLS} \\ \hat{\beta}^{WLS}_{-1} \\ \hat{\beta}^{WLS}_{C} \end{pmatrix} = \left(X^T W X\right)^{-1} X^T W Y,$$

Modeling

con r o o $\pi(p)$ c r on
r n o n on r o nc
p on n p o q n n
q on or o $R_n = \lambda[\pi(p_n)] = $ o $^{\pi(p_n)}$

TABLE 1 | Notations.

VARIABLES OR FUNCTIONS	
p	A generic value on the probability scale; objective probability or relative-frequency
$\lambda(\cdot)$	Log-odds function of probability or relative-frequency . $\lambda(p) = \log(p/(1-p))$
$\pi(p)$	Subjective probability or relative-frequency
p_n	Objective probability or relative-frequency of Trial n
S_n	Stimulus of Trial n in log-odds. $S_n = \lambda(p_n) = \log(p_n/(1-p_n))$
R_n	Response of Trial n in log-odds. $R_n = \lambda(\pi[p_n]) = \log(\pi[p_n]/(1-\pi[p_n]))$
MODEL ABBREVIATIONS	
LLO	Linear in log-odds model
AL	Adaptation-level model
LLO-L	Linear in log-odds model with linear sequential effects
LLO-NL	Linear in log-odds model with non-linear sequential effects
AL-L	Adaptation-level model with linear sequential effects
AL-NL	Adaptation-level model with non-linear sequential effects
MODEL PARAMETERS	
γ	Slope of the linear transformation of log-odds
p_0	Crossover point; controlling the intercept of the linear transformation of log-odds
β_0	Coefficient for the S_n term
β_{-i}	Coefficient for the R_{n-i} term, $i = 1,2,...,5$
β_C	Coefficient for the constant term
σ_{noise}	Standard deviation of the Gaussian noiseeation755(s)-0.367584(i)0.451604(a)0.2065455()0.2631895(o)-0.222301(e)0.20479788816(e)-278.55(r)-278.6713230Tc 206556w(i)0.1492732(r)18.19088(F)0.140088(l)0.40.4(n)0.177.86(e)0.ET0.5278.578(47(t)0.46735-0.399091(e740T723631(e)-2209((t)-279.592(o)-0.972.52w448

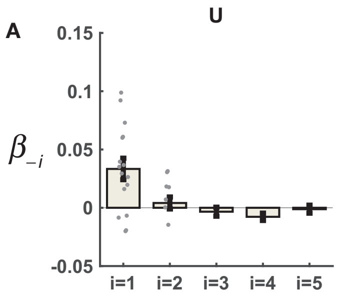
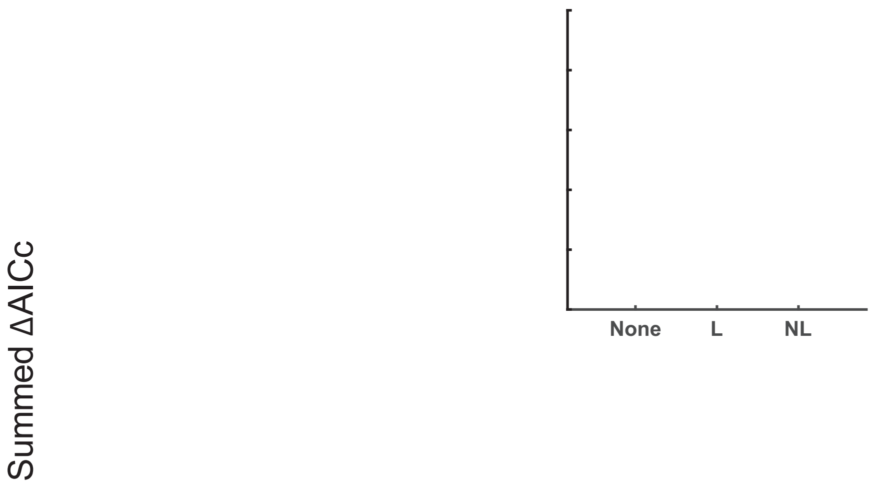

A
U
0.15
0.1
0.05
0
-0.05
β_{-i}
i=1
i=2
i=3
i=4
i=5

TABLE 2 | Pearson's r between data and model predictions in the pattern of sequential effects.

Condition	LLO	AL	LLO-L	AL-L	LLO-NL	AL-NL
Uniform	−0.015	−0.048	−0.215	−0.217	0.773	0.668
Extreme	−0.355	−0.174	−0.126	−0.318	0.723	0.588
Small	−0.141	0.219	−0.367	0.120	0.709	0.666
Large	0.238	−0.046	0.137	0.041	0.577	0.657

Model Comparison

Estimated Parameters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None
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NL

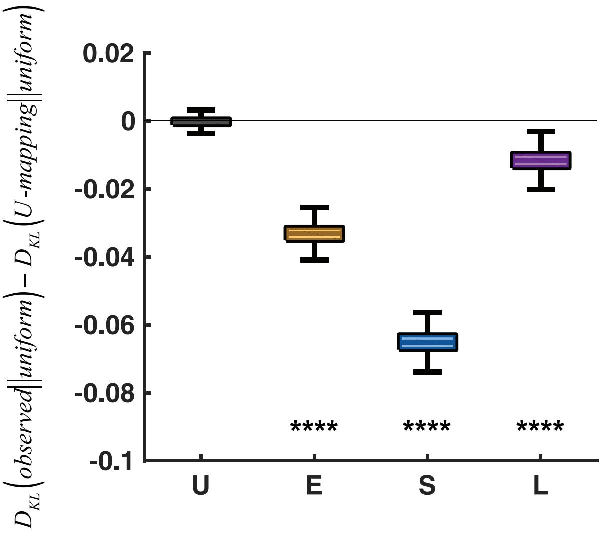

FIGURE 7 | Evidence for efficient coding**.** D_{KL} $(observed \| uniform)$ denotes the KL divergence from the uniform distribution over [0, 1] to the observed response distribution of a specific condition. D_{KL} $(U\text{-}mapping \| uniform)$ denotes the KL divergence from the uniform distribution to the response distribution predicted by the p-to-π (p) mapping of the Uniform condition. D_{KL} $(observed \| uniform) - D_{KL}$ $(U\text{-}mapping \| uniform)$ is plotted for each condition, with the Uniform condition serving as a sanity check (i.e., the difference should be 0). A negative difference implies that the p-to-π (p) mapping adopted by a specific condition is closer to efficient coding than applying the mapping of the Uniform condition to the condition. In the box plot, the middle line denotes the median across 1,000,000 simulations, the bottom and top lines denote the lower and upper quartiles, and the error bars denote the 99% confidence interval. ****

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[illegible] *NIPS'12 Proceedings of the 25th International Conference on Neural Information Processing Systems* [illegible]

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[illegible] *Proc. Natl. Acad. Sci. U.S.A.* [illegible]

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Conflict of Interest Statement: [illegible]